

Algebraic Geometry Session
XXIV Coloquio Latinoamericano de Álgebra
Organizers: Jorge Vitório Pereira and Giancarlo Urzúa

The talks of the SGA will be at the Ninoslav Bralic auditorium (Facultad de Matemáticas) from 14:00 to 17:00: Monday(4), Tuesday(4), Wednesday(3), and Thursday(3). Each talk will be **40 minutes** long. There will be available a black board or computer connections for your lecture.

Titles and Abstracts

On a problem of Gizatullin

Carolina Araujo (IMPA, Brazil)

In this talk, I will discuss the following question, attributed to Gizatullin: “Which automorphisms of a smooth quartic surface in projective 3-space are restrictions of Cremona transformations of the ambient space?” I will present a general framework that can be used to study this problem, namely the birational theory of Calabi-Yau pairs, and report on recent progress obtained in collaboration with A. Corti and A. Massarenti and D. Paiva and S. Zikas.

Counting polarizations on abelian varieties with automorphisms

Robert Auffarth (Universidad de Chile, Chile)

A natural question that arises when studying curves is the following: If two curves have Jacobians that are isomorphic as (non-polarized) tori, are the curves isomorphic? The answer to this question is a resounding no, with many counterexamples existing in the literature. The reason for this is the fact that Jacobians, and abelian varieties in general, can have many non-isomorphic principal polarizations. In 1987 Lange showed how to calculate the number of non-isomorphic principal polarizations on an abelian variety in terms of the automorphism group of the abelian variety and the Rosati involution induced by any principal polarization. In this talk we will show how to generalize Lange’s results to arbitrary (not necessarily principal) polarizations, and explain why this problem is interesting in the context of abelian varieties (and Jacobians) with non-trivial automorphisms that fix a given polarization.

Effective and Noether-Lefschetz divisors on orthogonal modular varieties

Ignacio Barros (University of Antwerp, Belgium)

I will survey important questions regarding modular forms and the divisor geometry of various moduli spaces such as those of K3 and hyperkähler varieties. In the second half, I will report on recent results about the structure of the NL-effective cone and the NL-positivity of the canonical divisor. This is a joint work in progress with P. Beri, L. Flapan, and B. Williams.

Tropical semirings and curve singularities

Ethan Cotterill (Universidade Estadual de Campinas, Brazil)

Curve singularities are classical objects of study in algebraic geometry. The key player in their combinatorial structure is the value semigroup, or its compactification, the value semiring. One natural problem is to explicitly determine the value semirings of distinguished infinite classes of singularities, with a view to understanding their asymptotic properties. In this paper, we establish a matroidal framework for systematically resolving this problem. More precisely, we show how to associate to any curve singularity a support semiring that maps homomorphically to the value semiring. This is a tropical semiring with a finitary matroid structure, and we show how its basic features explain well-known features of value semirings of singularities, including a natural characterization of minimal generating sets. In the case of either line arrangements (i.e., multiple points) or cusps, we can be more quantitatively precise; and our results have important consequences for the topology of Severi varieties of singular rational curves in projective space.

Likely Intersections

Sebastian Eterović (University of Leeds, UK)

The Zilber-Pink conjecture is a major open problem in arithmetic geometry which predicts that if V is a proper subvariety of an arithmetic variety S (e.g. abelian variety, Shimura variety, others) not contained in a proper “special” subvariety of V , then the “unlikely intersections” of V with the proper special subvarieties of S are not Zariski dense in V . In this talk I will present a strong counterpart to the Zilber-Pink conjecture, namely that under some natural conditions, likely intersections are in fact Euclidean dense in V . This is joint work with Thomas Scanlon.

Algebraic Montgomery–Yang problem and cascade conjecture

DongSeon Hwang (IBS, Center for Complex Geometry, South Korea)

Algebraic Montgomery–Yang problem predicts that a normal projective surface with quotient singularities has at most 4 singular points if its second Betti number is 1. This is an algebraic version of the original Montgomery–Yang problem concerning the number of exceptional orbits of pseudo-free circle actions on the five dimensional sphere. The algebraic Montgomery–Yang problem remains open when the surface is rational and the canonical divisor is ample. In this talk, we will briefly review the history of the problem and explain known results both in algebraic version and in the recent smooth version by Jo–Park–Park. We conclude the discussion by proposing a conjecture, called the cascade conjecture, which implies the algebraic Montgomery–Yang problem.

The Cox ring of an embedded variety

Antonio Laface (Universidad de Concepción, Chile)

Let X be a subvariety of a smooth projective toric variety Z . The inclusion $i: X \rightarrow Z$ induces homomorphisms at the level of divisor class groups $i_{Cl}: Cl(Z) \rightarrow Cl(X)$ and of Cox rings $i_R: R(Z) \rightarrow R(X)$. A natural question in this setting is how to compute a presentation for the Cox ring of X in terms of the image $R := Im(i_R)$. In this talk, I will show that if i_{Cl} is an isomorphism, then $R(X)$ is intersection of finitely many localizations of R , generalizing the results in “Cox rings and combinatorics II” by Hausen, “Hypersurfaces in Mori dream spaces” by Artebani and Laface, and “Birational geometry of hypersurfaces in products of projective spaces” by Ottem. As an application, we compute the Cox ring of all smooth Calabi-Yau hypersurfaces X in a smooth toric Fano variety Z of Picard rank 2 and dimension $\dim Z > 3$, showing that $R(X)$ is finitely generated a complete intersection. This is the joint work “The Cox ring of an embedded variety” with Luca Ugaglia and Cristóbal Herrera.

Irrationality and the Gauss map

Olivier Martin (IMPA, Brazil)

The degree of irrationality of a variety X is the minimal degree of a generically finite rational map from X to a projective space. This birational invariant measures how far X is from being rational. In the past decade, several obstructions which generalize classical obstructions to rationality have been used to obtain lower bounds on this invariant. I will discuss a new obstruction which uses the Gauss map of subvarieties of abelian varieties. As an application, we will show that the degree of irrationality of a genus 3 theta divisor is at least 4.

Automorphisms and Hodge structures of Klein hypersurfaces

Pedro Montero (Universidad Técnica Federico Santa María, Chile)

The Hodge structure of a smooth projective algebraic variety is an important invariant which, in many cases, is expected to determine the variety itself. This expectation is called the “Torelli principle”, and it is based on a classical theorem by Torelli for algebraic curves and their Jacobians. In this talk, we will focus on Klein hypersurfaces in the projective space, and we will explain how to determine their automorphism group and, in many cases, how to compute the automorphism group of the associated polarized Hodge structure. These results provide new positive evidence for the Torelli principle for cubic fivefolds and quartic threefolds, for which we can associate a principally polarized abelian variety called the Intermediate Jacobian. This is based on a joint work with Víctor González (UTFSM), Álvaro Liendo (U. de Talca) and Roberto Villaflor (UTFSM).

Butler’s conjecture for higher rank vector bundles over algebraic curves

Ángela Ortega (Humboldt University, Germany)

We consider semi-stable rank n vector bundles E on a smooth algebraic curve C . If E is globally generated by a linear subspace V of its sections, the kernel of the evaluation map $V \otimes \mathcal{O}_C \rightarrow E$ is a vector bundle $M_{E,V}$ of rank $n - \dim V$. Butler’s conjecture states that for the general curve C and the general pair (E, V) the vector bundle $M_{E,V}$ is semi-stable. This conjecture has obvious consequences for the higher rank Brill-Noether theory. It has been solved for line bundles and vector bundles of slope $> 2g - 2$. In this talk we review the known results on Butler’s conjecture, in particular for rank two bundles (joint work with L. Brambila-Paz, O. Mata-Gutierrez and P. Newstead) and we present the program to study the problem in any rank using stability conditions (joint work with S. Feyzbakhsh).

On the Torelli Theorem for graphs and stable curves

Marco Pacini (Universidade Federal Fluminense, Brazil)

The classical Torelli theorem states that a smooth curve can be recovered from its polarized Jacobian. In this talk, we will discuss the extensions of this theorem to stable curves and their dual graphs, and its dependence on the concept of compactified Jacobians. First we will recall Caporaso and Viviani’s theorem characterizing when two stable curves have the same Caporaso’s compactified Jacobian. Then we will discuss the Torelli theorem via fine compactified Jacobians. This is based on an ongoing joint work with Alex Abreu and Nicola Pagani.

Rational curve arrangements: geometry and combinatorics

Piotr Pokora (University of the National Education Commission Krakow, Poland)

In this talk we focus on the geometry of rational curve arrangements in various aspects of contemporary research. We explain their importance in the context of algebraic surfaces with high Picard numbers and the Numerical Terao's Conjecture. A number of instructive examples will be given.

A System of polynomial equations related to the Jacobian conjecture

Christian Valqui (Pontificia Universidad Católica del Perú, Perú)

We prove that the plane Jacobian conjecture is false if and only if there exists a solution to a certain system of polynomial equations. We analyse the solution set of this system. In particular we prove that it is zero dimensional.

Polarized endomorphism of Fano varieties with complements

José Yáñez (University of California Los Angeles, USA)

An endomorphism on a normal projective variety X is said to be polarized if the pullback of an ample divisor A is linearly equivalent to a qA , for some integer $q > 1$. Examples of these endomorphisms are naturally found in toric varieties and abelian varieties. Indeed, it is conjectured that if X admits a polarized endomorphism, then X is a finite quotient of a toric fibration over an abelian variety. In this talk, I will restrict to the case when X is a Fano type variety admitting a 1-complement, meaning that there exists an effective divisor B such that (X, B) is log Calabi-Yau, and $K_X + B \sim 0$. I will prove that if (X, B) has a polarized endomorphism that preserves the complement structure, then (X, B) is a finite quotient of a toric log Calabi-Yau pair. This is joint work with Joaquín Moraga and Wern Yeong.