XXIV Coloquio Latinoamericano de Álgebra
Sesión de topología algebraica

Organizadores: Jonathan Barmak y Mauricio Bustamante

1. Información general

La sesión será lunes, martes y miércoles de 14:00 a 17:00. Cada día tendremos 3 charlas de entre 45 y 50 minutos. Éstas empiezan a las 14:00, 15:00 y 16:00. Todos los ponentes hablan español, así que ese será el idioma preferido de las charlas. Sin embargo, queda a discreción del expositor dar su charla en español o en inglés.

Las charlas serán en la **Sala 2** de la Facultad de Matemáticas de la Universidad Católica de Chile (campus San Joaquín). Ésta cuenta con una pizarra de buen tamaño y proyector.

El orden de las charlas es el siguiente:

Lunes 22 de julio

Cubically presented groups, strong asphericity, and applications.

Macarena Arenas, University of Cambridge.

We'll explore the problem of finding effective models for the classifying spaces of certain quotients of fundamental groups of non-positively curved cube complexes, we'll discuss the framework – cubical small-cancellation theory – that provides the necessary tools to do so, and we'll explain how this viewpoint allows us to compute the homology and cohomology of various examples.

Cubulable relatively hyperbolic groups and virtual specialness.

Eduardo Reyes, Max Planck Institute, Bonn.

Cubulable groups are orbifold fundamental groups of compact non-positively curved cube complexes, and form a rich and particularly well-behaved class of CAT(0) groups. If a cubulable group is also word-hyperbolic, then Agol's theorem asserts that it is virtually special, a condition that guarantees the separability of many subgroups. This result was the final step in solving the virtual fibering and virtual Haken theorems for closed hyperbolic 3-manifolds, and also implies residual finiteness of many word-hyperbolic groups. I will talk about a generalization of Agol's theorem in the context of cubulable relatively hyperbolic groups, also obtained independently by Groves-Manning. We give a criterion for virtual specialness for such groups, which is satisfied when the peripheral subgroups are virtually abelian.

Quasimorphisms of big mapping class groups.

Israel Morales, Universidad de La Frontera.

The space $QM(G)$ of quasimorphisms of a group G, is defined as the kernel of the comparison map between the bounded cohomology group and the usual cohomology group in dimension 2. We will give a brief overview of the various applications of this concept in geometry, topology, and group theory. Additionally, I will attempt to convey the geometric ideas involved in calculating the space of quasimorphisms for certain classes of groups, including classical

mapping class groups. I will conclude the talk with the latest advances in the attempt to calculate the space $QM(G)$ for big mapping class groups.

Martes 23 de julio

Spherical complexes related to finite groups of Lie type.

Kevin Piterman, Philipps-Universität Marburg.

The Tits building associated with a connected reductive linear group G in positive characteristic p, can be identified with the order complex of the poset of parabolic subgroups. If G is simple and F is an endomorphism of G such that the fixed point set G^F is finite (i.e. some power of F is a power of the Frobenius map), then G^F is termed a finite group of Lie type in char p. Since G^F also admits a BN-pair, it has an associated building and a poset of parabolic subgroups, which are homotopy equivalent to the p-subgroup poset of G^F. In particular, they are homotopy equivalent to a wedge of spheres by the Solomon-Tits theorem, and the dimension of such spheres can be computed, roughly, from the action of F on the Dynkin diagram of G. In this talk, we will see that for a finite group H that contains a self-centralising normal subgroup L of Lie type in characteristic p, the p-subgroup poset of H is homotopy equivalent to a wedge of spheres of possibly different dimensions. We obtain this result by recovering the p-subgroup poset of H, up to homotopy equivalence, from the building of L after carefully gluing the "remaining" p-subgroups of H that intersect L trivially. In fact, this procedure allows us to obtain explicit formulas for the number of spheres in each dimension. As a corollary, we show that the Euler characteristic of such p-subgroup posets is non-trivial except in some cases where $p = 2$ and L is untwisted of type A_n ($n >= 2$) (i.e. a quotient of $SL(n + 1, p^a)$) or E_6. We also derive formulas for the H-module structure of the homology in terms of induced representations of Steinberg modules.

Morse theory of Bestvina-Brady type for posets and applications.

Gabriel Minian, Universidad de Buenos Aires.

I will present a Morse theory for posets of Bestvina-Brady type that combines matchings and height functions. This theory generalizes Forman's discrete Morse theory for regular CW-complexes and can be applied to all finite posets. Finally, I will discuss two applications of this theory: one related to Vietoris-Rips complexes and the other concerning the study of bounds for the rank of finitely presented groups.

On the powers of the Euler class for pure mapping class groups.

Rita Jiménez Rolland, Instituto de Matemáticas, UNAM, Oaxaca.

The mapping class group of an orientable closed surface of genus with one marked point can be identified, by the Nielsen action, with a subgroup of the group of orientation-preserving homeomorphisms of the circle. This inclusion pulls back the "discrete universal Euler class" producing a non-zero class in the second integral cohomology of the mapping class group. In this talk I will present an overview of what is known about the vanishing and non-vanishing behaviour of the powers of this class, including recent joint work with Solomon Jekel about the torsion of the -th power.

Miércoles 24 de julio

Critical ribbon knots.

José Ayala, Universidad de Tarapacá.

The main open problem in geometric knot theory is to develop a tabulation of knots based on an energy criterion, specifically aiming to present knots as global energy minimisers within their isotopy classes. These optimal configurations are sometimes referred to as ideal knots. The problem of computing exact values for the length functional, called ropelength for geometric knots in 3-space, and ribbonlength for geometric knot diagrams, has been resisting for some decades already. In this talk, we will discuss the spaces of ribbon knots and disk diagrams, and show how to obtain minimisers in spaces of these objects.

Fibred knots and quantum invariants.

Daniel López Neumann, Indiana University.

A knot in 3-space is said to be fibred if the complement of the knot admits a fibration into a circle where each fiber is a surface with boundary the knot. The topological invariants of knots coming from the "quantum world", such as the Jones polynomial, HOMFLY or Khovanov homology, are typically unrelated to this property. On the other hand, an old theorem states that the Alexander polynomial (from classical algebraic topology) of a fibred knot is monic. In this talk, we will show that the "evil cousins" of the Jones polynomial (a.k.a non-semisimple quantum invariants) are actually well-behaved with respect to fibredness: they are q-monic on fibred knots. The proof is quite simple and geometric, we will only rely on formal properties of the given quantum invariants so no background on these will be needed.

Monodromy groups of quadratic differentials.

Rodolfo Gutiérrez, CMM-Universidad de Chile.

The moduli space of Riemann surfaces carrying quadratic differentials has been extensively studied in recent years. However, not much is known about its topology and, in particular, its (orbifold) fundamental group G. A simpler problem is understanding the image of a natural monodromy representation of G into a mapping class group or a symplectic group. In this talk, I will discuss what is known and what is open about this problem.