

Computational Algebra
XXIV Coloquio Latinoamericano de Álgebra
Organizers: Mercedes Pérez, and Nelly Villamizar.

Schedule

Monday

1400 Carlos Valencia (Cinvestav-IPN)

1430 Rodrigo Contreras (UChile)

1520 Alonso Sepúlveda (UFU)

1550 Juliana García Galofre (UDESA)

Tuesday

1400 Gabriela Jeronimo (UBA)

1430 Nicolas Thériault (USACH)

1520 Sebastián Higuera (UNAL)

1550 Andrés Rubiano (UNAL)

Titles and Abstracts

A minimal free resolution of the duplication of an ideal monomial

Carlos E. Valencia Oleta (cvalencia@math.cinvestav.edu.mx, Cinvestav-IPN, Mexico).

Inspired by how vertex duplication affects the edge ideal of a graph, we will introduce the duplication I^\diamond of a monomial ideal I . We will describe a multigraded minimal free resolution of I^\diamond in terms of a resolution for I

The action of the automorphism group on the Weierstrass points of hyperelliptic surfaces: algorithm, properties, applications Rodrigo Contreras (r.andres.c.martinez@gmail.com, UChile, Chile).

The use of representation theory is very common in the field of complex geometry. For a hyperelliptic Riemann surface X the rational representation ρ_H for its automorphism group $G = \text{Aut}(X)$ yields information about the action of G on the first group of homology of the surface $H^1(X, \mathbb{Z})$.

The set of Weierstrass points W is, usually, hard to determine for a generic curve, but for a hyperelliptic surface it suffices to compute the set of fixed points of the hyperelliptic involution σ

$$W = \text{Fix}(\sigma)$$

The action of G on W yields a different representation ρ_W which, for hyperelliptic surfaces, is orthogonal to ρ_H but unfortunately it is not faithful.

$$\ker(\rho_W) = \{id, \sigma\}$$

In this talk we will show an algorithm used to compute ρ_W and its character using ρ_H as a reference.

Pure Gaps and Codes in Function Fields

Alonso Sepúlveda Castellanos (alonso.castellanos@ufu.br, UFU, Brazil).

Abstract: In this work, we provide a way to completely determine the set of pure gaps $G_0(P_1, P_2)$ at two rational places P_1, P_2 in a function field F over a finite field \mathbb{F}_q , and its cardinality. As a consequence, we completely determine the set of pure gaps and its cardinality for two families of function fields: the *GK* function field and Kummer extensions. We apply these results to construct AG codes.

This is joint work with E. R. Mendoza (UFRJ), L. Quoos (UFRJ) and G. Tizziotti (UFU).

Beyond Boolean networks, a multivalued approach

Juliana García Galofre (jgarcialgalofre@udesa.edu.ar, UDESA, Argentina).

Genes can have several modes of action, depending on their expression levels. Binary variables are often not sufficiently rich, requiring the use of multi-valued networks instead. Our approach is based on a representation of multi-valued networks using multi-valued logic functions, providing a biologically intuitive representation of the network.

The steady state analysis of Boolean networks is computationally complex, and increasing the number of variable values beyond 2 adds substantially to this complexity, and no general methods are available beyond simulation. We show a set of reductions and an algorithm to compute the steady states of a multi-valued network that has a complexity that, in many cases, is essentially the same as that for the case of binary

values. Furthermore, it uses tools to compute lattice points in rational polytopes, tapping a rich area of algebraic combinatorics as a source for combinatorial algorithms for Boolean network analysis.

This is joint work with A. Dickenstein, A. Galarza Rial, R. Laubenbacher and M. Pérez Millán.

Bounds for the geometric degree of the tangent bundle of a smooth algebraic variety

Gabriela Jeronimo (jeronimo@dm.uba.ar, UBA-CONICET, Argentina).

The degree of an algebraic variety is a fundamental invariant which turns out to be a key parameter involved in the complexity of algorithms for solving polynomial equations. In this talk, we will discuss the relation between the geometric degree of the tangent bundle of a smooth irreducible affine algebraic variety and the degree of the given variety. Questions of this kind appear naturally in the analysis of complexity aspects in symbolic approaches to the integrability of systems of differential algebraic equations and in the effective differential Nullstellensatz.

We will present general upper bounds for the degree of the tangent bundle in terms of the degree of the variety that are exponential in the dimension or the codimension of the variety. In addition, we will show particular situations in which we obtain linear or quadratic bounds, including the cases of parametric curves and varieties defined by generic polynomial equations.

This is joint work with Leonardo Lanciano and Pablo Solernó (Universidad de Buenos Aires, Argentina).

Speeding up the computation of group order of genus 2 over finite fields

Nicolas Thériault (nicolas.theriault@usach.cl, USACH, Chile).

In 2012, Gaudry and Schost obtained the first known randomly selected cryptographically interesting hyperelliptic curve of genus 2 over a field of 127 bits. Producing this curve required a very significant computation, in the order of one million CPU-hours, in which computing the order of a specific curve (after a comparatively fast selection process) would take on average close to 1000 CPU-hours. Furthermore, the curves considered had full 2-torsion groups, i.e. the curve has group order $16 \cdot \text{prime}$.

We present new developments in the computation of the group order of specific curves which allow us to significantly reduce the cost of several of the sub-algorithms used by Gaudry and Schost, specifically: improved division-by-L algorithms, specialized factorization algorithms and the use of special towers of field extensions, which allow us to compute the group order of each hyperelliptic curve in less than 100 CPU-hour. Furthermore, our algorithm applies to more general curves and we are able to obtain various examples of curves with prime order (giving slightly stronger curves) over fields of 127 bits.

Weak annihilators and nilpotent associated primes of noncommutative rings

Sebastián David Higuera Rincón (sdhiguarar@unal.edu.co, UNAL, Colombia).

In this talk, we will mention some definitions, remarks, and important results of the theory of weak annihilators and nilpotent associated prime ideals over Ore extensions, following the work of Ouyang and Birkenmeier. Later, we will present the skew PBW extensions introduced by Gallego and Lezama and discuss the results obtained concerning the weak annihilators and nilpotent associated prime ideals for this family of noncommutative rings. This work extends the theory to different families of algebras such as Auslander-Gorenstein rings, some Calabi-Yau and skew Calabi-Yau algebras, some Artin-Schelter regular algebras, quantum universal enveloping algebras, families of differential operator rings, and many other algebras of great interest in noncommutative algebraic geometry and noncommutative differential geometry. The talk is based on joint work with Armando Reyes.

Noncommutative differential geometry of quantum planes

Andrés Alejandro Rubiano Suárez (arubianos@unal.edu.co, UNAL, Colombia).

During the last decades, many properties of this geometry have been studied. In fact, there are many open problems in Riemannian geometry, the solution of which is being strongly sought by many geometricians today. For this reason, it can be said that Riemannian geometry is one of the branches with the most influence and the most work in mathematics. However, many of these problems are sought starting from classical Riemannian geometry (here, classical will refer to the commutative one). At this point we want to study a generalization of classical Riemannian geometry, to a non-commutative one; or, as it is known in the literature, a quantum Riemannian geometry. In this talk, we will present some algebras that have differential geometry through automorphisms and derivations.