GEOMETRÍA COMPLEJA

Calabi-Yau complete intersections in toric varieties

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Most of the known examples of Calabi-Yau (CY) varieties are constructed as complete intersections in toric varieties. The toric embedding allows, on one side, to control the singularities of these varieties and, on the other side, to define different versions of mirror symmetry in a combinatorial way. Examples of these are the Batyrev-Borisov duality [2] for complete intersection CY in toric Fano varieties and the Berglund-Hübsch-Krawitz [3] duality for hypersurfaces in weighted projective spaces. In this talk we will provide a sufficient condition, for a complete intersection in a toric variety, to define a CY variety. Moreover, we use this result to propose a version of Berglund-Hübsch-Krawitz duality for complete intersections. As an application, we find dual families for codimension two complete intersection K3 surfaces in weighted projective spaces.

This is a joint work with Paola Comparin and Robin Guilbot, based on [1].

References

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Translation covers of platonic solids

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Starting with a Platonic solid, we can produce a Riemann surface that carries a natural Abelian differential. These surfaces have been studied from several viewpoints. In this talk, I will discuss their geometric and dynamical properties.

General Coherent Systems
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Maps from a curve X to projective spaces are studied in terms of linear systems, i.e., pairs (L, V), consisting of a line bundle L on X and a linear subspace $V \subset H^0(X, L)$. These pairs were studied within classical Brill–Noether theory. In order to investigate, e.g., moduli spaces of semistable vector bundles on X, one looks at higher rank *Brill–Noether pairs* or *coherent systems*. These are pairs (E, V) composed of a vector bundle E on X and a linear subspace $V \subset H^0(X, E)$. It is natural to fix the rank r and the degree d of E as well as the dimension s of V. Unlike the case of line bundles, i.e., $r = 1$, it is necessary to introduce a stability condition and restrict to the corresponding semistable coherent systems in order to obtain reasonable moduli spaces for coherent systems of type (r, d, s). Brill–Noether theory for higher rank vector bundles turned out to be substantially different from Brill–Noether theory for line bundles, and the basic questions have not yet been settled in full generality.

Now, let us fix a reductive group G, a finite dimensional vector space V, and a representation $\varrho\colon G \longrightarrow \mathrm{GL}(V)$ of G on V. Then, we may associate with any principal G-bundle $\mathcal{P}$ on X a vector bundle $\mathcal{P}_\varrho$ on X with typical fiber V. Let us also fix a line bundle L on X. Then, a general coherent system consists of a principal G-bundle $\mathcal{P}$ on X and a subspace $V \subset H^0(X, \mathcal{P}_\varrho \otimes L)$. For $G = \mathrm{GL}(r)$, $\varrho = \mathrm{Ad}_G$ the adjoint representation, and $L = \omega_X$ the canonical line bundle, the study of these objects has been proposed by Brambila-Paz, García-Prada, and Gothen. In this case, it is expected that the resulting moduli spaces will have applications to the geometry of moduli spaces of Higgs bundles. Another interesting case arises for $G = \mathrm{GL}(r)$, ϱ a symmetric power of the standard representation, and L an arbitrary line bundle. The resulting objects are linear systems of divisors in projective bundles over X.

In the talk, I will propose a notion of semistability for general coherent systems and present results on the existence of moduli spaces and the Kobayashi–Hitchin correspondence, the latter by Cesare Goretti, for $G = \mathrm{GL}(r)$, ϱ a tensor power of the standard representation, and L the trivial line bundle.[1]

This work grew out of discussions with Leticia Brambila-Paz at the XXII Coloquio Latinoamericano de Álgebra in Quito, Ecuador. It has been supported by the DFG project "New developments in the moduli theory of decorated sheaves".

[1]This case does not include coherent Higgs systems which are being studied by Edgar Castañeda, a PhD student of Leticia Brambila-Paz.

Linear stability of coherent systems on curves

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The notion of linear stability of a variety in projective space was introduced by Mumford in the context of GIT. It has subsequently been applied by Mistretta, Stoppino and others to Butler's conjecture on stability of the dual span bundle (DSB) of a generated linear series (E, V), where E is a rank one vector bundle over a smooth curve and V is a subspace of global sections of E that generates E. In this talk we recall some results on linear stability for linear series on curves. We then extend the definition of linear stability to generated coherent systems of higher rank, that is, for (E, V) where E is a higher rank vector bundle over a smooth curve. We give examples of coherent systems with unstable DSB that are also linearly unstable. We show that linearly stable coherent systems of rank 2 with $\dim(V) = 4$ and $\deg(E) = d$ for low enough d have stable DSB, and use this to prove a particular case of Butler's conjecture. We then exhibit a linearly stable generated coherent system with unstable DSB, confirming that linear stability of (E, V) in general remains weaker than semistability in higher rank.

This is part of a joint work with George H. Hitching from Oslo Metropolitan University.

On Riemann surfaces completely determined by the order of their automorphism group

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In this talk, we consider compact Riemann surfaces that are uniquely determined by the property of possessing a group of automorphisms of a prescribed order, strengthening uniqueness results proved by Nakagawa [1], who dealt with cyclic automorphism groups. More precisely, we deal with the cases in which such an order is $3g$ and $3g+3$, where g is the genus. We prove that if g is odd (respectively g even and $g \not\equiv 2 \mod 3$) then there exists a unique Riemann surface of genus g with a group of automorphisms of order $3g$ (respectively $3g+3$), and such an automorphism group is necessarily cyclic. Also, we prove that such a uniqueness result may fail if one takes out the hypothesis g odd (respectively, g even) by providing infinite families of counterexamples. Finally, we determine the full automorphism group of such Riemann surfaces and provide decompositions of their Jacobians.

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Grupos que se realizan como el grupo de automorfismos de un origami

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Una clase especial de superficies de traslación son aquellas que son cubriente del toro unitario ramificado sobre el origen. Estos son llamados origamis o superficies teseladas por cuadrados. En el caso de origamis compactos, se han estudiado extensivamente. Los origamis no compactos, por otro lado, han sido estudiados muy poco y, por lo regular, los resultados generales a los que se puede aspirar es cuando se estudian clases especiales de origamis. En esta charla contaremos las ideas que nos llevaron a establecer que todo grupo numerable se puede ver como el grupo de automorfismos de un origami. Trabajo en conjunto con Rubén A. Hidalgo.

Variedades abelianas definidas sobre cuerpos finitos, estructura de grupo y aplicaciones

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En esta presentación hablaremos sobre la estructura de grupo de los puntos racionales de variedades abelianas definidas sobre cuerpos finitos. En particular, analizaremos aquellas variedades cuyo grupo de puntos racionales es un grupo cíclico, daremos resultados recientes obtenidos y algunos proyectos futuros. También expondremos algunas relaciones de la teoría de variedades abelianas definidas sobre cuerpos finitos y su relación con la teoría de códigos.

Ends space of the fiber product over infinite-genus Riemann surfaces

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Considering non-constant holomorphic maps $\beta_i : S_i \to S_0$, $i \in \{1, 2\}$, between non-compact Riemann surfaces for which it is associated its fiber product $S_1 \times_{(\beta_1,\beta_2)} S_2$. With this setting, in this talk we relate the ends space of the singular surface $S_1 \times_{(\beta_1,\beta_2)} S_2$ to the ends space of its normal fiber product $S_1 \widetilde{\times_{(\beta_1,\beta_2)}} S_2$. Moreover, we provide conditions on the maps β_1 and β_2 to guarantee connectednes of the surface $S_1 \times_{(\beta_1,\beta_2)} S_2$. From these conditions, we link the ends space of the surface $S_1 \times_{(\beta_1,\beta_2)} S_2$ with the topological type of the Riemann surfaces S_1 and S_2. Finally, we study the fiber product over infinite hyperelliptic curves and discuss its connectedness and ends space [1].

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One Dimensional Families of Riemann Surfaces with Automorphisms

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The moduli space $\mathcal{M}_g$ of surfaces of genus $g \geq 2$ is the space of conformal equivalence classes of closed Riemann surfaces of genus g. This space is a complex, quasi-projective variety of dimension $3g-3$, and geometrically an orbifold under the action of the modular group. The set of singular points, the *branch locus*, may be stratified into a finite, disjoint union of smooth, irreducible, quasi-projective subvarieties called *equisymmetric strata*. Each stratum corresponds to a collection of surfaces of the same *symmetry type*, i.e. the conjugacy class of a finite group in the modular group.

The (complex) 1D strata are Riemann surfaces with punctures. We study these strata as punctured Riemann surfaces, in terms of the action of the automorphism group of the surfaces in the stratum. In this way the strata can be constructed, a la Hurwitz, as (branched) coverings of the Riemann sphere with four punctures or a torus with one puncture.

Dessins d'enfants y álgebra de superpotenciales

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Sea $D = (X, \mathscr{G})$ un dessin d'enfant, es decir, X es una superficie compacta y orientable, y $\mathscr{G} \subset X$ es un grafo bipartito finito tal que $X - \mathscr{G}$ es la unión de discos topológicos [5,6]. En esta charla determinaremos la función de Belyi [2,5,7] y álgebra superpotencial para árboles [3,10] y curvas de género uno [4]. Además, presentaremos un algoritmo para calcular el álgebra superpotenciales para un dessin d'enfant en términos de su grupo de monodromía y aplicaremos nuestro algoritmo implementado en Magma [8] y Phyton para determinar el álgebra superpotencial para árboles de grado 11 [4]. Finalmente, presentamos polinomios de Shabat y su relación con los pares de Belyi [1,9], y determinamos dicho polinomio para algunos árboles.

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