XXIV CLA – Session: Hopf Algebras and Tensor Categories

Schedule

Monday

14 hs Cristian Vay (Universidad Nacional de Córdoba, Argentina).

15 hs Julia Plavnik (Indiana University, EEUU).

16 hs Vitor Ferreira (Universidade de São Paulo, Brasil).

Tuesday

14 hs Daniel López Neumann (Indiana University, EEUU).

15 hs Fabio Calderon (Universidad de Los Andes, Colombia).

16 hs Dirceu Bagio (Universidade Federal de Santa Catarina, Brasil).

Wednesday

14 hs Steen Ryom-Hansen (Universidad de Talca, Chile).

15 hs José Simental (Universidad Nacional Autónoma de México, México).

Titles and abstracts

Dirceu Bagio (Universidade Federal de Santa Catarina, Brasil)

The Green ring of the Hopf algebra $\mathfrak{u}(\mathfrak{m})$

Let $\Bbbk$ be an algebraically closed field of characteristic 2 and $\mathfrak{s}$ the unique, up to isomorphism, not restricted simple Lie algebra of dimension 3 over $\Bbbk$ which has basis $\{e, h, f\}$ and bracket $[e, f] = h$, $[e, h] = e$ and $[f, h] = f$. The 2-closure $\mathfrak{m}$ of $\mathfrak{s}$ (that is, $\mathfrak{m}$ is the smallest restricted Lie algebra that contains $\mathfrak{s}$) is a 5-dimensional Lie algebra and its restricted enveloping algebra $\mathfrak{u}(\mathfrak{m})$ is generated by e, f, h with defining relations

$$ef + fe = h, \qquad eh + he = e, \qquad fh + hf = f, \qquad e^4 = f^4 = 0, \qquad h^2 + h = 0.$$

It was proven in [1] that $\mathfrak{u}(\mathfrak{m})$ is special biserial and hence it is of tame representation type [2]. Also, we proved in [1] that the finite-dimensional indecomposable $\mathfrak{u}(\mathfrak{m})$-modules are string or band modules. In this work in progress, we reinterpret these families of indecomposable modules as syzygy, cosyzygy and (k,k)-type modules. This new approach allows us to calculate the Green ring of $\mathfrak{u}(\mathfrak{m})$. This is a joint work with N. Andruskiewitsch, S. D. Flora and D. Flôres.

References

[1] N. Andruskiewitsch, D. Bagio, S. Flora and D. Flôres. *On the Drinfeld double of the restricted Jordan plane in characteristic 2.* arXiv:2303.02228

[2] W. Crawley-Boevey. *Tameness of biserial algebras.* Arch. Math. **65**, 399-407 (1995).

Fabio Calderon (Universidad de Los Andes, Colombia)

Classification of graded Hopf algebra quotients

Let G be a group. A Hopf algebra H is called G-graded if H is G-graded as an algebra, and the grading is compatible with the comultiplication, counit and antipode. Examples of such Hopf algebras include cocentral extensions of Hopf algebras and the twisted Drinfeld double of groups. In this talk, we present a classification of Hopf ideals for a G-graded (quasi-)Hopf algebra based on the following parametrization: normal subgroups N of G, Hopf ideals in the homogeneous component of the identity

H_e that are invariant under N, and G-equivariant trivializations of a specific quotient constructed with these parameters. This approach incorporates ideas from earlier work by César Galindo and Corey Jones, who parameterized all fusion subcategories arising from equivariantization through a group action on a fusion category. However, in our results, the Hopf algebras are not necessarily semisimple, and G is not necessarily finite. This talk is based on joint work with César Galindo.

Vitor Ferreira (Universidade de São Paulo, Brasil)

Braid group actions on quantum invariants of free algebras

Given a finite-dimensional module V over a finite-dimensional Hopf algebra H, the tensor algebra $T(V)$ becomes a module algebra with a linear action by H. It is known that the algebra of invariants $T(V)^H$ of the action of H on $T(V)$ is always free, but very rarely finitely generated. However, taking into account the action of the symmetric groups by place permutations on its homogeneous components, it can be finitely described, when H is cocommutative and semisimple, as show by Koryukin in 1994. In the present work, we present evidence to support that the same happens if H is taken to be quasi-triangular and the symmetric groups are replaced by the braid groups. This work is the result of a collaboration with Lucia Murakami and Lucas Ogawa and was partially funded by FAPESP (Projeto Tematico 2020/16594-0).

Daniel López Neumann (Indiana University, EEUU)

Non-semisimple tensor categories and geometric topology

It is well-known that braided tensor categories lead to topological invariants of knots in 3-space. The Jones polynomial is an example, where the category is that of representations of quantum $sl(2)$ at generic q. However, what kind of geometric information of knots do these invariants capture? At least for the Jones polynomial, this is the subject of various conjectures. In this talk, I'll discuss the common trend of various recent works with Roland van der Veen: certain non-semisimple tensor categories produce knot invariants that contain much clearer geometric information (e.g. minimal genus of Seifert surfaces, fibrations). I'll explain this through the case of quantum $sl(2)$ at a root of unity. No background on geometric topology is needed.

Julia Plavnik (Indiana University, EEUU)

Exact factorizations and bicrossed product of fusion categories

Finding new examples of fusion categories is important for the advancement of its theory. One way to look for them is by considering new constructions. In this talk, we will start by presenting the definition of exact factorization of fusion categories and some of their properties and examples. We will first consider the fusion ring of an exact factorization. Then we will introduce the notion of a matched pair of fusion categories and use it to define the bicrossed product of fusion categories. We will show the relation of this new construction with exact factorizations. This talk is based on joint work with M. Müller and H.M. Peña Pollastri.

Steen Ryom-Hansen (Universidad de Talca, Chile)

The KLR-approach to the representation theory of the Temperley-Lieb algebra and the bt-algebra

This talk is partially based on results obtained in collaboration with K. Ormeño. The KLR-algebras were introduced 10-15 years ago, by Khovanov-Lauda and independently by Rouquier. In the talk we first explain some of their wonderful properties. We then focus on some applications to the representation theory of the Temperley-Lieb algebra and of the bt-algebra in knot theory.

José Simental (Universidad Nacional Autónoma de México, México)

Calibrated representations of the double Dyck path algebra

The double Dyck path algebra $\mathbb{B}_{q,t}$ is an associative, non-unital algebra defined by Carlsson and Mellit to aid in their proof of the shuffle conjecture. For every $n \geq 0$, $\mathbb{B}_{q,t}$ contains a copy of the extended affine Hecke algebra $\mathcal{H}_q(n)$, together with operators linking $\mathcal{H}_q(n)$ and $\mathcal{H}_q(n+1)$. Following work of Grojnowski, Ram, Vazirani, ... for affine Hecke algebras, we define the notion of a calibrated representation of $\mathbb{B}_{q,t}$, and construct several examples of these using both geometric and combinatorial methods. We show that, under suitable conditions, one can define tensor products and duals of calibrated representations. Time permitting, I will elaborate on connections to the representation theory of Ding-Iohara type algebras, such as the elliptic Hall algebra. This is joint work with Nicolle González and Eugene Gorsky.

Cristian Vay (Universidad Nacional de Córdoba, Argentina)

Linkage principle for small quantum groups

We call "small quantum groups" the Drinfeld doubles of bosonizations of finite-dimensional Nichols algebras. These include Lusztig's small quantum groups associated with root systems of Cartan type but there are also small quantum groups with root systems of super and modular type, among others. In Lie Theory, the linkage principle gives a necessary condition for a simple module being a composition factor of a Verma module. In this talk we will show such a principle for our Drinfeld doubles by adapting techniques from the work of Andersen, Jantzen and Soergel in the context of Lusztig's small quantum. As a consequence, we will find a notion of (a)typicality similar to the one in the representation theory of Lie superalgebras. The typical simple modules turn out to be the simple and projective Verma modules. Moreover, we deduce a character formula for 1-atypical simple modules. This talk is based on the preprint `arXiv:2310.00103`.