Homological Methods, CLA 2024

Talk schedule

	Tuesday	**Wednesday**
2:00	Eugenia Ellis	Jason McCoullough
2:35	Sinem Odabasi	Edgar Velasco
3:10	Maria Souto	Jorge Guccione
3:45 - 3:55	break	break
3:55	Alejandro Argudin	Emmanuel Jerez
4:30	Martha Lizbeth Shaid Sandoval Miranda	Valente Santiago

- Presentation: 25 minutes
- Q&A: 5 minutes
- Transition to next speaker: 5 minutes

Talk Abstracts

Homotopy structures realizing algebraic kk-theory

Eugenia Ellis
Instituto de Matemática y Estadística
"Prof. Ing. Rafael Laguardia"
Facultad de Ingeniería - UDELAR

Abstract:
Algebraic kk-theory, introduced by Cortiñas-Thom, is a bivariant K-theory defined on the category of algebras over a commutative unital ring ℓ. It consists of a triangulated category kk endowed with a functor $j : \mathrm{Alg} \to kk$ that is the universal excisive, homotopy invariant and matrix-stable homology theory. Moreover, we can recover Weibel's homotopy K-theory from kk since we have $kk(\ell, A) = KH(A)$ for any algebra A. In this talk we will see that the category of algebras with fibrations the split surjections and weak equivalences the kk-equivalences is a category of fibrant objects, whose homotopy category is kk. Using this, we are able to construct a stable infinity category whose homotopy category is kk. This is a work in progress, joint with Emanuel Rodríguez Cirone.

Homotopy category of R-module valued additive functors

Sinem Odabaşı*
Departamento de Matemáticas
Universidad de Murcia
Murcia, España

Abstract

The homotopy and derived category of a ring are the main subjects of study in (Categorical) Homological Algebra. Both are obtained from the category of chain complexes of a ring, which is a particular case of functor categories. In [1], the authors show that under certain conditions on a small k-linear category Q, there exists a 'Q-shaped derived category' of a ring R, which is obtained by the projective/injective model structure on the functor category $k\text{-Lin}(Q, R\text{-Mod})$. In this talk, we discuss if $k\text{-Lin}(Q, R\text{-Mod})$ equipped with the object-wise trivial exact structure is a Frobenius category, which yields a kind of 'Q-shaped homotopy category of R.' The conditions imposed on Q would be subtly relaxed compared to those given in [1], and therefore, we could apply it to different kinds of categories. The talk contains certain results from a joint work with Sergio Estrada and Manuel Cortes Izurdiaga.

References

[1] Holm, Henrik; Jorgensen, Peter, *The Q-shaped derived category of a ring*, Journal of the London Mathematical Society **106(4)**, (2022). 3263-3316.

*Partially supported by grant I+D+i PID2020-113206GB-I00, funded by MICIU/AEI/10.1303/501100011033 and by grant 22004/PI/22 funded by Fundación Séneca, e-mail: `sinem.odabasi@um.es`

The category of periodic complexes

María José Souto Salorio

Universidade da Coruña, España
e-mail: maria.souto.salorio@udc.es

This is a joint work with Claudia Chaio, Alfredo Gonzalez and Isabel Pratti.

Let $\mathcal{A}$ be an additive category and $\mathbf{C}^b(\mathcal{A})$ the category of bounded complexes. We denote by $\mathbf{C}_{\equiv m}(\mathcal{A})$ the category consist of the m-periodic complexes over $\mathcal{A}$ and by $\mathbf{K}_{\equiv m}(\mathcal{A})$ its relative homotopy category. These categories have independent interest by itself but also they are related to the compression functor and orbit categories.

In case $\mathcal{A} = \mathbf{K}^b(\operatorname{proj} A)$, for a finite dimensional algebra of finite global dimension, the compression functor $F_m : \mathbf{K}^b(\operatorname{proj} A) \to \mathbf{K}_m(\operatorname{proj} A)$ induces an embedding of the orbit category $\mathbf{K}^b(\operatorname{proj} A)/[m]$ into its triangulated hull. Therefore, the orbit category inherits a triangle structure from $\mathbf{K}^b(\operatorname{proj} A)$ if and only if F_m is dense. Several authors studied these categories and look for conditions under which F_m is dense (see for example [1], [2], [3], [4]).

In this talk, we present some properties concerning the category of $m-$periodic complexes $\mathbf{C}_{\equiv m}(\mathcal{A})$ in case that $\mathcal{A}$ is a dualizing category. In particular, we show conditions under which the compression functor $\mathcal{F}_m : \mathbf{C}^b(\mathcal{A}) \to \mathbf{C}_{\equiv m}(\mathcal{A})$ is dense and other related ideas.

References

[1] B. Keller. *On triangulated orbit categories.* Doc. Math. 10, (2005), 551–581.

[2] L. Peng, J. Xiao. Root Categories and Simple Lie Algebras. *Journal of Algebra.* 198: 19–56 (1997).

[3] S. Saito. *Tilting theory for periodic triangulated categories.* Math. Z. 304, 47 (2023)

[4] T. Stai. The triangulated hull of periodic complexes. *Math. Res. Lett 25,* 1: 199–236 (2018).

Categories of quiver representations and relative cotorsion pairs

Alejandro Argudin Monroy
Facultad de Ciencias, Universidad Nacional Autónoma de México

Abstract:
Given a (possibly infinite) quiver Q and an abelian category A, we study the category $\mathrm{Rep}(Q, A)$ of representations of Q with values in A and the full subcategory of finite support representations $\mathrm{Repf}(Q, A)$. Namely, we will revisit some constructions made by Enochs, Estrada, Garcia Rozas, Holm, Jorgensen and Odabasi. We will show conditions on Q and A in order to determine whether $\mathrm{Rep}(Q, A)$ and $\mathrm{Repf}(Q, A)$ have enough projectives and to express the global dimension of $\mathrm{Rep}(Q, A)$ and $\mathrm{Repf}(Q, A)$ in terms of the global dimension of A. Lastly, we will show conditions on Q and A in order to perform a construction of a cotorsion pair in $\mathrm{Rep}(Q, A)$ and $\mathrm{Repf}(Q, A)$ from a cotorsion pair in A. This is joint work with Octavio Mendoza.

An overview of preradical theory in some categories

Dra. Martha Lizbeth Shaid Sandoval Miranda
Departament of Mathematics, Office AT-242
Universidad Autónoma Metropolitana,
Iztapalapa-Campus, Mexico City, CDMX,
marlisha@xanum.uam.mx
https://sites.google.com/izt.uam.mx/marlisha

Let R be an associative ring with a unit element. A preradical σ on $\mathrm{R-Mod}$ is a subfunctor of the identity functor $\mathrm{Id}\colon \mathrm{R{-}Mod} \to \mathrm{R{-}Mod}$. Over the years, the study of preradicals has been a cornerstone in the study of torsion and localization theories. In Mexico, the work carried out for decades by the group started by F. Raggi, J. Ríos, H. Rincón and R. Fernández-Alonso can be highlighted. Recently, G. A. López-Cafaggi studied preradicals in semiabelian categories and torsion network theories in simplicial groups [4], while S. Pardo-Guerra and G. Silva applied the theory of preradicals to study how information flows through certain structures [3].

In this talk we will see results obtained jointly by *R. Fernández-Alonso (UAM-Iztapalapa), J. Magaña Zapata (UAM-Azcapotzalco) and V. Santiago-Vargas (F. Ciencias UNAM)*, on generalizations to Abelian categories of classical preradical results from the lattice and categorical theoretical point of view, [1].

Research supported by UAM-PROJECT ITAC ("Interactions between Topology, Algebra and Categories"), and CONAHCYT-PROJECT CBF2023-2024-2630.

References

[1] Fernández-Alonso R., Magaña-Zapata J., Santiago V., Sandoval-Miranda M.L.S. *Galois conections and preradicals in Abelian Categories.* (Submitted). Preprint 2024.

[2] Medina-Bárcenas M., Sandoval-Miranda M.L.S. , Zaldívar-Corichi A. *A point-free version of torsionfree classes and the Goldie torsion theory.* ArXiv:2402.17084 https://doi.org/10.48550/arXiv.2402.17084

[3] Pardo-Guerra S., Silva G. A. *On preradicals, persistence, and the flow of information.* International Journal of General Systems, 1–25. (2024) https://doi.org/10.1080/03081079.2024.2348665

[4] López-Cafaggi G. *Torsion theories of simplicial groups with truncated Moore complex.* ArXiv:2202.07067. https://doi.org/10.48550/arXiv.2202.07067

Koszul Graded Moebius Algebras

Jason McCullough
Iowa State University

Abstract:
The graded Moebius algebra of a matroid is a commutative graded algebra which encodes the combinatorics of the lattice of flats of the matroid. As a special subalgebra of the augmented Chow ring of the matroid, it plays an important role in the recent proof of the Dowling-Wilson Top Heavy Conjecture. Koszul algebras are special graded, quadratic algebras with wonderful duality properties. Recently, Mastroeni and McCullough proved that the Chow ring and the augmented Chowring of a matroid are Koszul. We study when graded Moebius algebras are Koszul. We characterize the Koszul graded Moebius algebras of cycle matroids of graphs in terms of properties of the graphs. Our results yield a new characterization of strongly chordal graphs via edge orderings. This is joint work with Adam LaClair, Matthew Mastroeni, and Irena Peeva.

CLA2024
XXIV Coloquio Latinoamericano de Álgebra
22-26 July 2024

Title of the talk: Hochschild-Mitchell Cohomology of triangular matrix categories.

Abstract: In this talk we will discuss the Hochschild-Mitchell Cohomology of categories of triangular matrices. Given a triangular matrix category Λ = (T 0 M U), we establish the Hochschild-Mitchell cohomology relation H^{i}(Λ) and H^{i}(U) of Λ and U respectively; and show that there exists an exact long sequence relating them, for which we make use of homological epimorphisms in functor categories. This result extends the well-known Cibils-Michelana-Platzeck result.

Extensions of Linear Cycle Sets and Cohomology

Jorge A. Guccione
Facultad de Ciencias Naturales - UBA

Abstract:
The celebrated Yang–Baxter equation was first established by Yang in 1967 and then by Baxter in 1972, motivated by problems in quantum physics. Since then, many solutions of various forms of the Yang–Baxter equation have been constructed by physicists and mathematicians. The relevance of this equation in mathematics and physics and the fact that the construction of new solutions is widely open, led Drinfeld to ask for the family of set-theoretical solutions. An important class of set-theoretical solutions of the Yang-Baxter equation are the non-degenerate involutive solutions. The study of these solutions led to the introduction of the following mathematical structures: bijective 1-cocyles, braces and linear cycle sets (which, in fact, are different avatars of the same notion). In this talk I will use mainly the language of linear cycle sets, which are abelian groups $(A, \cdot)$, endowed with another binary operation satisfying suitable conditions. This structure was introduced by Rump. I will talk about the general notion of extensions of linear cycle sets (particular cases were considered by Bachiller, Lebed-Vendramin and Nir Ben David-Ginosar). Finally, I will talk about a cohomology, whose second cohomology group classifies the extensions of a linear cycle set H by a trivial linear cycle set I. This generalizes the main results obtained by Lebed and Vendramin. This talk is based on a work carried out jointly with Juan José Guccione and Christian Valqui.

EMMANUEL JEREZ

The partial group homology

Partial group actions are a generalization of group actions and naturally appear when we focus our attention on the local behavior of a symmetry. If we have an action of a group G on a space X, focusing on the symmetry structure of a subspace Y of X generally does not yield a group action; instead, we obtain what we call a partial group action of G on Y. Alongside the concept of partial group action, we have the concept of partial representations of a group G. The category of partial representations is isomorphic to the category of $K_{par}G$-modules, where $K_{par}G$ is a certain universal algebra that governs the partial representations of G.

In 2017, E. R. Alvares, M. Muniz Alves, and M. J. Redondo introduced the partial group cohomology $H^{\bullet}_{par}(G, M)$ of a group G with coefficients in a $K_{par}G$-module. This cohomology theory is a generalization of the usual group cohomology that appear as a component of a spectral sequence that converges to the Hochschild cohomology of a partial crossed product algebra $\mathcal{A} \rtimes G$.

In this talk, we will also consider the homological version of the aforementioned cohomology and answer the question: what is the relationship between partial group (co)homology and the usual group (co)homology? We will show that partial group homology is naturally isomorphic to the usual group homology, and by dualizing this result, we obtain a spectral sequence for the cohomological framework that collapses in several cases.

1 E. Jerez, *On the homology of partial group actions*, (preprint).

Homological Theory of Idempotent Ideals in Functor Categories

Valente Santiago Vargas
Facultad de Ciencias, UNAM

Abstract:
In this talk, we will explain the theory of k-idempotent ideals in the setting of dualizing varieties. Several results given previously by M. Auslander, M. I. Platzeck, and G. Todorov are extended to this context. Given an ideal I (which is the trace of a projective module), we construct a canonical recollement which is analogous to a well-known recollement in categories of modules over Artin algebras. Moreover, we study the homological properties of the categories involved in such a recollement.