XXIV CLA – SESSION: REPRESENTATION THEORY

ORGANIZERS: JOHN MACQUARRIE AND HIPOLITO TREFFINGER

Dalia Artenstein *Universidad de la República*

Title: The Hochschild cohomology ring of monomial path algebras

Abstract: In this talk, I will discuss a joint work with Janina Letz, Amrei Oswald, and Andrea Solotar.

The Hochschild cohomology of an associative algebra over a field k has the structure of a graded commutative k-algebra with the cup product. It has been shown that this structure is zero in positive degrees for quadratic triangular string algebras [Bustamante 2006] and triangular string algebras [Redondo-Roman 2014]. In this work, we generalize these results by proving that for triangular monomial algebras the cup product is always zero in positive degrees. To do this, we describe the diagonal map associated to Bardzell's resolution, providing a method for computing the cup product for any monomial algebra.

Raymundo Bautista *Universidad Nacional Autónoma de México – Morelia*

Title: Generic Modules and Homological Systems

Abstract: Let Λ be a finite dimensional k-algebra with k algebraically closed field. A finite homological system (Mendoza-Saenz-Changchang Xi [2]) consists of a finite preordered set $(\mathcal{P}, \leqslant)$ and a family of pairwise non-isomorphic indecomposable finitely generated Λ-modules $\{\Delta_i\}_{i\in P}$ with the following conditions.

(1) $\mathrm{Hom}_\Lambda(\Delta_i, \Delta_j) \neq 0$ implies $i \leqslant j$.
(2) $\mathrm{Ext}^1_\Lambda(\Delta_i, \Delta_j) \neq 0$ implies $i \leqslant j$ by $j \not\leqslant i$.

We recall that a preordered set $(\mathcal{P}, \leqslant)$ is a non-empty set equipped with a reflexive and transitive relation $\leqslant$. Take $\Delta = \{\Delta_i\}_{i\in\mathcal{P}}$. Denote by $\mathcal{F}(\Delta)$ the full subcategory of the finitely generated Λ-modules whose objects are 0 and the finitely generated Λ-modules M, for which there is a filtration of submodules:

$$0 = M_t \subset M_{t-2} \subset \cdots \subset M_1 \subset M_0 = M$$

such that each M_j/M_{j+1} is isomorphic to some $\Delta_i \in \Delta$. A generic Λ-module G is a Λ-module having the following properties:

(i) G is indecomposable.
(ii) $\dim_k G$ is infinite:
(iii) G has finite length as a $\mathrm{End}_\Lambda(G)^{op}$-right module.

In [1] it has been proved that $\mathcal{F}(\Delta)$ is either tame or wild but not both. We will see that in the case $\mathcal{F}(\Delta)$ is of tame representation type there are generic Λ-modules G with a filtration of submodules:

$$0 = G_t \subset G_{t-2} \subset \cdots \subset G_1 \subset G_0 = G$$

such that each G_j/G_{j+1} is isomorphic to an infinite coproduct of copies of some $\Delta_i \in \Delta$, we call them Δ-generic modules. These generic module controls the morphisms of must of the indecomposable modules in $\mathcal{F}(\Delta)$. For instance given a dimension d, there are Δ generic modules $G_1, \ldots, G_n$ such that $End(G_i)$ is a field for all $i = 1, \ldots, n$ if and only if for almost all indecomposable $M \in \mathcal{F}(\Delta)$, with $\dim_k M = d$, $\mathrm{End}_\Lambda(M) \cong k$.

This is a joint work with Efrén Pérez and Leonardo Salmerón.

References:

1. R. Bautista, E. Pérez , L. Salmerón. Tame and wild theorem for the category of modules filtered by standard modules. Journal of Algebra 650 (2024) 394-457
2. O. Mendoza, C.Sáenz, Ch. Xi. Homological systems in module categories over preordered sets. Q. J. Math 60(1) (2009) 75-103

Claudio Rodriguez Beltrán *Universidad Nacional de Colombia*

Title: On biequipped posets and their representations

Abstract: Biequipped posets and their representations are introduced. A biequipped poset is a poset with two additional relations contained in the order relation that satisfy an additional condition. Every finite biequipped poset has an associated $\mathbb{R}$-incidence algebra of matrix type, which as a ring is semiperfect and of right-peak, see [2]. For each finite biequipped poset, a set of representations is defined on the triple $(\mathbb{R}, \mathbb{C}, \mathbb{H})$, where $\mathbb{R}$ and $\mathbb{C}$ are the fields of real and complex numbers respectively and, $\mathbb{H}$ is the division ring of the quaternions. Objects are collections of right F-linear subspaces (submodules) (F varying between $\mathbb{R}$, $\mathbb{C}$ or $\mathbb{H}$) of a finite-dimensional $\mathbb{H}$-linear main space, and morphisms are $\mathbb{H}$-linear maps between the main spaces that respect the order and additional relationships.

A finite representation type criterion is discussed, it can be deduced directly from [1] and some examples of indecomposable representations of biequipped posets are presented as an example. A matrix representation of a biequipped poset is introduced in order to begin the study of biequipped posets of infinite representation type following the ideas of [3].

References:

1. V. Dlab & C.M. Ringel, *On Algebras of Finite Representation Type.* Journal of Algebra, **33**, (1975), 306–394.
2. B. Klemp & D. Simson, *Shurian sp-representation-finite right peak PI-rings and their indecomposable socle projective modules.* Journal of Algebra, **134**, (1990), 390–468.
3. C. Rodríguez & A.G. Zavadskij, *On corepresentations of equipped posets and their differentiation*, Revista Colombiana de Matemáticas, **41**, (2007), 117–142.

Felipe Gallego Olaya *Universidad de Antioquia*

Title: Relations between the strong global dimension, complexes of fixed size and derived categories

Abstract: Let $\mathbb{Z}$ be the integer numbers, $\Bbbk$ an algebraically closed field, Λ a finite dimensional $\Bbbk$-algebra, modΛ the category of finitely generated right Λ-modules, projΛ the full subcategory of modΛ consisting of all projective objects, and $\mathrm{C}_n(\mathrm{proj}\Lambda)$ the bounded complexes of projective Λ-modules of fixed size for an integer $n \geq 2$. We describe an algorithm to calculate the strong global dimension of Λ, when Λ has finite strong global dimension and is derived-discrete, by using the Auslander-Reiten quiver of the category $\mathrm{C}_n(\mathrm{proj}\Lambda)$. Moreover, we also show the relationship between the Auslander-Reiten quiver of the bounded derived category $\mathrm{D}^b(\mathrm{mod}\Lambda)$ and the Auslander-Reiten quiver of $\mathrm{C}_{\eta+1}(\mathrm{proj}\Lambda)$, where η is the strong global dimension of Λ.

Ana García Elsener *Universidad Nacional de Mar del Plata*

Title: Skew Brauer graph algebras

Abstract: Brauer graph algebras are defined combinatorially through a graph with some additional data on their vertices, given by a function called multiplicity function and an order of the edges attached to each vertex. They appeared first in representation theory of groups, and were defined by Donovan and M.R. Freislich to classify indecomposable modules over quasi-Frobenious algebras.

A result by Schroll says that each Brauer graph algebra with multiplicity function identically one is the trivial extension of a gentle algebra. Skew-gentle algebras were defined by Geiss and De la Peña to study a subclass of matrix-problems. These algebras were also studied by V. Bekkert, E. Marcos and Merklen They generalize the class of gentle algebras as they are, by definition, skew-group algebras of gentle algebras.

In this project we define skew-Bruauer-graph algebras, a family that generalizes Brauer graph algebras. We prove that each skew-Brauer-graph algebra with multiplicity function identically one is the trivial extension of a skew-gentle algebra. We classify skew-Brauer graph algebras of finite representation type. We identify these skew-Bruauer-graph algebras with unbordered compact Riemann surface dissections, and see skew gentle algebras and their reflections as operations that add or change boundary pieces to the surface.

This was a joint project with Victoria Guazzelli (UNMdP Argentina) and Yadira Valdivieso (U. Puebla Mexico)

Hernán Giraldo *Universidad de Antioquia*

Title: Auslander-Reiten triangles in Frobenius categories and Applications

Abstract: Let $(\mathcal{C}, \mathcal{E})$ be a Krull-Schmidt Frobenius category, that is, $\mathcal{C}$ is an additive category, $\mathcal{E}$ is a class of exact pairs, and $(\mathcal{C}, \mathcal{E})$ is an exact category. For $\underline{\mathcal{C}}$, the stable category of $\mathcal{C}$, we prove that every Auslander-Reiten triangle in $\underline{\mathcal{C}}$, is induced from a special Auslander-Reiten sequence in $\mathcal{C}$ when these exist. This result generalizes the same result given by Y. Calderón-Henao, H. Giraldo, and J.A. Vélez-Marulanda in [1], that is, they obtained this result for $\widehat{\Lambda}$-$\underline{\text{mod}}$ the stable category of $\widehat{\Lambda}$-mod the abelian category of finitely generated left $\widehat{\Lambda}$-modules, where $\widehat{\Lambda}$ is the repetitive algebra of Λ the finite dimensional $\Bbbk$-algebra with $\Bbbk$ an algebraically closed field. As an application of our result we obtain an easy proof, than the one given by E.R. Alvares, S.M. Fernandes, and H. Giraldo in [2], of how to get the shape of the Auslander-Reiten triangles in the bounded derived category $\mathrm{D}^b(\mathrm{mod}\Lambda)$, with $\mathrm{mod}\Lambda$ being the category of finitely generated right Λ-modules. Finally, we noted how our result is applied to give a relation between the strong global dimension, complexes of fixed size, and the bounded derived category $\mathrm{D}^b(\mathrm{mod}\Lambda)$, this relations was given by Y. Calderón-Henao, F. Gallego-Olaya, and H.Giraldo (preprint).

This is a joint-work with Yohny Calderón-Henao and Felipe Gallego-Olaya.

References:

1. Calderón-Henao, Y., Giraldo, H. and Vélez-Marulanda J.A. *On irreducible morphisms and Auslander-Reiten triangles in the stable category of modules over repetitive algebras*, Springer, Algebr Represent Theor, Volume **26**, (2023), 2039-2055.
2. Alvares, E.R., Fernandes, S. M., and Giraldo H., *Shape of the Auslander-Reiten triangles*, Algebras and Representation Theory, Volume **23**, Number 6, (2020), 2257-2274.

Viviana Gubitosi *Universidad de la República*

Title: Coloured mutation of coloured quivers of type $\mathbb{A}_n$

Abstract: In this talk we will define the coloured mutation of coloured quivers. Coloured quivers and its respective mutation were introduce by Buan and Thomas in his work [1]. This generalize the well known quiver mutation of Fomin and Zelevinsky [2].

A colored quiver Q is a finite quiver whose arrows are associated with a color c in $\{0, \ldots, m\}$, and satisfies the following properties: Q has no loops, and it adheres to two additional properties called monochromaticity and skew-symmetry. Every acyclic quiver Q can be viewed as a colored quiver by coloring each arrow with color 0 and adding an arrow of color m in the opposite direction.

Buan and Thomas also defined an operation on the set of colored quivers called colored mutation, such that when there is only one color ($m = 1$), colored mutation coincides with the mutation defined by Fomin and Zelevinsky. The aim of this talk is to describe in a purely combinatorial manner the class of colored quivers that arise from mutating a quiver of type $\mathbb{A}_n$.

This work is a collaboration with Rafael Parra and Claudio Qureshi.

References:

1. A. Buan, H. Thomas, *Coloured quiver mutation for higher cluster categories.* Adv. Math. 222(3):971-995, 2009.
2. S. Fomin, A. Zelevinsky, *Cluster algebras I: Foundations*, J. Amer. Math. Soc. 15 (2): 497-529, 2002.

Mindy Y. Huerta *Universidad Nacional Autónoma de México*

Title: Quotient categories with exact structure from $(n+2)$-rigid subcategories in extriangulated categories.

Abstract: In this talk we introduce the notion of higher $\mathbb{E}$-extension groups for an extriangulated category $\mathcal{C}$ and study the quotients $\mathcal{X}^{\vee}_{n+1}/[\mathcal{X}]$ and $\mathcal{X}^{\wedge}_{n+1}/[\mathcal{X}]$ when $\mathcal{X}$ is an $(n+2)$-rigid subcategory of $\mathcal{C}$. We also prove (under mild conditions) that each one is equivalent to a suitable subcategory of the category of functors of the stable category of $\mathcal{X}^{\wedge}_{n+1}$ and the co-stable category of $\mathcal{X}^{\vee}_{n+1}$, respectively. Moreover, it can be induced an exact structure through these equivalences and we analyze when such quotients are weakly idempotent complete, Krull-Schmidt or abelian. The above discussion is also considered in the particular case of an $(n+2)$-cluster tilting subcategory of $\mathcal{C}$ since in this case we know that $\mathcal{X}^{\wedge}_{n+1} = \mathcal{C} = \mathcal{X}^{\vee}_{n+1}$. Finally, by

considering the category of conflations of a exact category, we show that it is possible to get an abelian category from these quotients. This is a joint work with O. Mendoza, C. Sáenz and V. Santiago.

Leo Margolis *Universidad Autónoma de Madrid*

Title: Units in group rings and representation theory of groups of prime order

Abstract: The unit group $U(\mathbb{Z}G)$ of the integral group ring $\mathbb{Z}G$ of a finite group G is a long studied object, but nevertheless many fundamental questions remain open. The strongest possible expectation on finite subgroups of $U(\mathbb{Z}G)$, expressed by Zassenhaus, had been that these always lie inside the trivial units $\pm G$ up to conjugation in the bigger group algebra $\mathbb{Q}G$. While this has been refuted in this most general form, it is still open for p-subgroups, and also whether the order of elements in $U(\mathbb{Z}G)$ coincide with those in G remains unknown (after a suitable normalization process).

I will explain a method which has been used to achieve several results on these questions, but the bottle neck of which, for the moment, is the understanding of representations of the most simplest groups one can imagine - the cyclic groups of prime order p. Overcoming some of these difficulties, recent progress allows us to show that Zassenhaus' question has a positive answer at least for units of order p, when the Sylow p-subgroup of G is also assumed to be of prime order.

This is joint work with Florian Eisele.

Jeremy Rickard *University of Bristol*

Title: Injective generation of the derived category

Abstract: The unbounded derived category of modules for a ring is generated, as a triangulated category with coproducts, by the projective modules. In 2001, Keller raised the question of whether it was also generated in the same sense by the injective modules. For general rings there are counterexamples, but for finite-dimensional algebras over a field the question is still open, and a positive answer would imply the Finitistic Dimension Conjecture, a major open question in homological algebra. I will discuss some of the recent activity around this question.

Pedro Jesus Hernandez Rizzo *Universidad de Antioquia*

Title: Universal deformation rings of a special class of modules over generalized Brauer tree algebras

Abstract: Deformation theory of finitely generated modules over a finite-dimensional algebra is a consolidated area in representation theory of algebras which was motivated by Mazur's deformation theory for finite-dimensional representations of profinite groups. More precisely, F.M. Bleher and J. A. Vélez-Marulanda developed a deformation theory for finite-dimensional algebras by showing that every module over a self-injective algebra with stable endomorphism ring isomorphic to $\Bbbk$ admits a universal deformation ring. Besides, in the case of Frobenius algebras, they showed that the universal deformation rings of modules are preserved under the syzygy functor Ω. Hence, an interesting problem is the classification of universal deformation rings of modules M over a Frobenius algebra Λ provided that the stable endomorphism ring is isomorphic to $\Bbbk$. A special class of Frobenius algebras are the symmetric algebras and an even more special class of these are the Brauer graph algebras. Brauer graph algebras come from modular representation theory of finite groups and appear in several contexts of representation theory. Furthermore, there exist important researches in universal deformation rings of modules over Brauer graph algebras. For example, by F. M. Bleher and D. Wackwitz determine the universal deformation rings of finitely generated modules over Brauer graph algebras of finite-representation type, that is, Brauer tree algebras. In the same spirit, the work by D. Meyer, R. Soto and D. Wackwitz address the problem in the infinite-representation type case. They compute the universal deformation rings of modules over generalized Brauer tree algebras of polynomial growth. In both works, derived equivalences are used. Derived equivalence of Brauer graph algebras coming from Kauer moves and other criteria like Opper-Zvonareva's derived equivalences classification which are useful to reduce the class of algebras in consideration to a simpler case.

In this talk, we will present an extension of the results by Meyer et al. to generalized Brauer tree algebras with non-polynomial growth in the case of periodic string modules. We compute the universal deformation rings of periodic string modules M over such algebras whose stable endomorphism ring is

isomorphic to $\Bbbk$. We use derived equivalences involving a star by applying Opper-Zvonareva's classification of derived equivalences of Brauer graph algebras. Finally, for a generalized Brauer tree algebra, we consider components of type $\mathbb{Z}_\infty^\infty$. Using the derived equivalence associated with the star, we prove that there exists a certain number of Ω-stable components in the stable Auslander-Reiten quiver composed entirely by modules with universal deformation ring isomorphic to $\Bbbk$ or $\Bbbk[[x]]/(x^2)$.

This is a joint work with Jhony Caranguay-Mainguez and José Vélez-Marulanda.

Sonia Trepode *Universidad Nacional de Mar del Plata*

Title: Skew group algebras and its applications to skew gentle algebras

Abstract: Gentle Algebras play an important role in representation theory of finite dimensional k-algebras. Recently the skew gentle algebras have caught the attention of several researchers.

In this talk we will discuss some properties and applications of skew group algebras. We study split extensions of skew gentle algebras, some examples are trivial extensions algebras, relation extensions algebras and partial relations extensions algebras. In order of study skew algebras of relation extension algebras we focus in admissible cuts of relation extensions algebras. We obtain a characterization, in terms of admissible cuts, of which algebras produce the same relation extension algebra. On the other hand, we obtain invariants as Hochschild cohomology of skew gentle algebras and representation dimension of skew gentle algebras.

Joint work with Yadira Valdiveso-Diaz, Universidad de las Américas, Puebla.