CLA2024, SESIÓN TEORÍA DE NÚMEROS

1. Títulos y abstracts

(1)
- Ariel Pacetti (Universidad de Aveiro, Portugal)
- **Título**: Galois representations of Algebraic Hypergeometric Motives
- **Resumen**:

  In a very nice article, Schwartz gave a complete list of rational parameters (a, b, c) for which the function ${}_2F_1(a, b, c)$ is algebraic. It is expected the existence of a 1-parameter, 2-dimensional Galois representation attached to each entry of his list, whose image is a finite group (related to the Galois group of the hypergeometric function). Assuming standard modularity conjectures, the representation should be related to a Hilbert modular form of parallel weight 1. In this talk we will prove that this is indeed the case when the monodromy group is solvable, by studying the attached Euler curve. To ease the exposition, we will focus on a particular tetrahedral case.

(2)
- Claudio bravo (École Polytechnique, Francia)
- **Título**: Sobre ciertos grupos homología relativa de subgrupos aritméticos de SU(3)
- **Resumen**: Un objeto clásico de estudio en geometría aritmética son los grupos algebraicos lineales sobre cuerpos globales. Ejemplos de esto son los grupos clásicos de matrices GLn, SL_n o Sp_{2n} definidos sobre cuerpos de números o sus análogos en característica positiva.

  Los subgrupos aritméticos de grupos algebraicos en caracteística positiva son de particular interés en aritmética. Ejemplo de dichos grupos son $\mathrm{GL}_n(\mathbb{F}_{p^r}[t])$ o $\mathrm{SL}_n(\mathbb{F}_{p^r}[t, t^{-1}])$. Dichos grupos pueden ser estudiados mediante la Teoría geométrica de grupos. En una primera parte de esta charla presentaremos algunos resultados clásicos sobre las aplicaciones de dicha teoría al estudio de la homología de grupos aritméticos.

  En la segunda parte, nos enfocaremos en estudiar la homología de ciertos subgrupos aritméticos G del grupo (no escindido, cuasi escindido) $\mathcal{G} = \mathrm{SU}_3$ definido por una forma hermitiana (isotropica) en 3 variables. Más precisamente, describiremos los grupos de homología relativa de G módulo un sistema de representantes $\mathfrak{U}$ de las clases de conjugación de sus subgrupos maximales unipotentes. En otras palabras, mediremos cuán diferentes son los grupos de homología de G de la suma directa de los respectivos grupos de homología de los subgrupos $U \in \mathfrak{U}$.

(3)
- David Grimm (USACH, Chile)
- **Título**: Rigidities in dual graphs of an arithmetic curve and sums of squares of rational functions

- **Resumen**: We introduce and study a certain symmetry concept in the dual graph of an arithmetic curve that we call "rigidities" for the time being. We obtain quantitative estimates on them that then allow us to improve on a well known genus-inequality for reductions of an arithmetic curve, taking into consideration arithmetic properties of each irreducible component of the reduction, such as the non-existence of a rational or formally real point in the normalization of each component.

  By applying this to curves over the field of iterated real formal series, we obtain an (optimal) bound on the second Pfister-index of such curves (the index of the group of sums of two squares inside the totally positive multiplicative group of the curve's function field). This bound had been previously known only in the case of hyperelliptic curves (by methods that are specific to hyperelliptic curves). This is joint ongoing work with Gonzalo Manzano (postdoc UChile).

(4)
- Gonzalo Tornaría (U de la República, Uruguay)
- **Título**:
- **Resumen**:

(5)
- Gustavo Rama (U. de la República, Uruguay)
- **Título**: Orthogonal modular forms for $O(5)$, paramodular forms, and congruences.
- **Resumen**: We introduce orthogonal modular forms, particularly modular forms for the group $O(5)$. Along with Dummigam, Pacetti, and Tornaría, combined with a work by Rösner and Weissauer, we proved that certain of these forms correspond to Siegel modular forms invariant under the paramodular group. We also proved several examples of Harder's conjecture, which relates Hecke eigenvalues of classical modular forms to Hecke eigenvalues of paramodular forms. Additionally, we proved a conjectured congruence by Buzzard and Golyshev between a modular form of weight 2 and level 61 and the non-lift paramodular form of weight 3 and level 61. Together with Assaf, Ladd, Tornaría, and Voight, we computed databases of paramodular forms for weights $(k, j) = (3, 0), (4, 0), (3, 2)$ and levels less than 1000. We search for new congruences of Harder type and of Buzzard, Golyshev type. We will show new examples of these congruences and provide proofs for some of them.

(6)
- Harald Helfgott (CNRS-IMJ, Francia)
- **Título**:Expansion, divisibility and parity
- **Resumen**:Hablaremos de un grafo que describe cuales enteros son divisibles por cuales primos. Probaremos que este grafo tiene una propiedad de expansión local fuerte casi en todas partes. Obtendremos luego diversos resultados en la teoría de números, allende de la barrera de paridad, aplicando esta propiedad de expansión junto con el conocido resultado de Matomäki-Radziwill

sobre las medias de funciones multiplicativas en los intervalos pequeños. Por ejemplo: para lambda la función de Liouville (la función completamente multiplicativa con $\lambda(p) = -1$ para todo primo p), $(1/\log x)\sum_{n\leq x}\lambda(n)\lambda(n+1)/n = O(1/\sqrt{(}\log\log x))$, lo cual es más fuerte que las conocidas cotas de Tao y Tao-Teraväinen (que cuentan con logaritmos cuádruples y triples, respectivamente), amén de varios otros resultados.

Discutiremos también una generalización reciente por C. Pilatte, quien, siguiendo una variante de la misma estrategia, ha logrado probar que un grafo similar con aristas que corresponden a enteros sin factores primos pequenos (mientras que en el grafo original las aristas corresponden solo a números primos). Como consecuencia, ha obtenido una cota de $O(1/(logx)^c)$ en vez de $O(1/sqrt(loglogx))$ para la suma más arriba, así como varias mejoras en otras consecuencias.

(7)
- Hector del Castillo (USACH, Chile)
- **Título**:On the Langlands functoriality principle and Ramanujan Conjecture
- **Resumen**:Langlands functoriality principle is one of the far-reaching conjectures of the Langlands program. It establishes a connection between automorphic representations of two groups that share certain structural data. In this talk, we are going to study this principle and present some cases of this conjecture and how to apply it to obtain a version of the Ramanujan Conjecture.

(8)
- Lola Thompson (U. Utrech, Países Bajos)
- **Título**:Preimágenes de la función de suma de divisores propios
- **Resumen**: Sea $s(n)$ la suma de los divisores propios de un número entero n. La función $s(n)$ ha sido estudiada durante miles de años, debido a su conexión con los números perfectos. En 1992, Erdős, Granville, Pomerance y Spiro (EGPS) conjeturaron que si $\mathcal{A}$ es un conjunto de números enteros con densidad asintótica cero, entonces $s^{-1}(\mathcal{A})$ también tiene densidad asintótica cero. Esto ha sido confirmado para ciertos conjuntos específicos $\mathcal{A}$, pero permanece abierto en general. En esta charla, daremos un panorama del progreso reciente hacia la conjetura EGPS. Nos centramos, en particular, en preimágenes de conjuntos a los que les faltan dígitos.

  Esta charla se basa en un trabajo conjunto con Paul Pollack y Carl Pomerance, y también en un trabajo conjunto con Kübra Benli, Giulia Cesana, Cécile Dartyge y Charlotte Dombrowsky.

(9)
- Lucas Villagra (Universidad Nac. de Cordoba, Argentina)
- **Título**: Endomorphism algebras of GL_2-type abelian varieties and Diophantine applications.
- **Resumen**: Let f and g be two newforms with equal coefficient fields. In this talk we will see how a congruence between the Galois representations of f and g for a sufficiently large prime

p implies strong conditions between the algebras of endomorphisms of the abelian varieties associated to the newforms f and g. Then, we will see one of the possible applications of such a result for the proof of the non-existence of solutions of Diophantine equations.

(10)
- Luis Palacios (U de Chile)
- **Título**: Geometría de la variedad propia de Bianchi alrededor de puntos no cuspidales.
- **Resumen**: Sea K un cuerpo cuadrático imaginario y p un número primo. La variedad propia de Bianchi es un espacio de moduli de formas automorfas p-ádicas de $\mathrm{GL}_{2/K}$. En esta charla presentaré nuestros resultados sobre la geometría local de la variedad propia de Bianchi alrededor de puntos no cuspidales. Este es un trabajo conjunto con Daniel Barrera.

(11)
- María Chara (Researcher of CONICET at Universidad Nacional del Litoral, Argentina)
- **Título**:Towards a classification of cyclic AG-codes
- **Resumen**:Let $\mathbb{F}_q$ be a finite field with q elements. A linear code of length n and dimension k over $\mathbb{F}_q$ is simply an $\mathbb{F}_q$-linear subspace $\mathcal{C}$ of $\mathbb{F}_q^n$ of dimension $\dim \mathcal{C} = k$.
Among the classical linear codes over $\mathbb{F}_q$, the family of cyclic codes have shown to be important and widely used because of their good parameters, excellent detection-correction capabilities and fast and efficient encoding-decoding algorithms. A code $\mathcal{C}$ is *cyclic* if it is closed under cyclic permutations of the coordinates of its codewords. That is, for any $c = (c_1, c_2, \ldots, c_n) \in \mathcal{C}$, the cyclic shift $s(c) = s(c_1, c_2, \ldots, c_n) = (c_2, \ldots, c_n, c_1)$ is also in $\mathcal{C}$. There is a natural action of the symmetric group $\mathbb{S}_n$ on $\mathbb{F}_q^n$ defined as $\tau(a_1, \ldots, a_n) = (a_{\tau(1)}, \ldots, a_{\tau(n)})$ for $(a_1, \ldots, a_n) \in \mathbb{F}_q^n$, $\tau \in \mathbb{S}_n$. This action defines the so called permutation automorphism group $\mathrm{PAut}(\mathcal{C})$ of $\mathcal{C}$ as the subgroup of $\mathbb{S}_n$ preserving $\mathcal{C}$, that is

$$\mathrm{PAut}(\mathcal{C}) = \{\tau \in \mathbb{S}_n : \tau(\mathcal{C}) = \mathcal{C}\}.$$

We see at once that a code $\mathcal{C}$ is cyclic if and only if the n-cycle $\sigma = (12\cdots n) \in \mathrm{PAut}(\mathcal{C})$. Clearly the above mentioned cyclic shift s corresponds to the n-cycle $\sigma = (12\cdots n)$. It is worth to mention that the family of cyclic codes contains important codes such as Golay codes, binary Hamming codes, Reed-Solomon codes and BCH codes.
A major breakthrough in coding theory was given by Goppa at the beginning of the 80's when he introduced a whole new family of linear codes obtained by evaluation of rational functions on rational points of an irreducible and smooth projective curve over $\mathbb{F}_q$. These codes are known today as algebraic geometry codes (or AG-codes for short). Using Goppa's ideas and modular curves over $\mathbb{F}_q$, Tsfasman et al constructed a family of codes which surpassed the Gilbert-Varshamov bound for the very first time. Thus, it seems natural to consider cyclic algebraic

geometry codes because, in this way, we will have the above mentioned advantages of cyclic codes combined with the conceptual richness involved in the construction of AG-codes.

In this talk, we will present a construction of cyclic algebraic geometry codes in the context of algebraic function fields over a finite field by using their group of automorphisms. We will also show that after a detailed study of the monomial equivalence of cyclic algebraic geometry codes constructed with our method, there is only one non-equivalent code in the case of a rational function field.

This talk is based on a joint work with Gustavo Cabaña (UNL), Ricardo Podestá (UNC) and Ricardo Toledano (UNL).

(12)
- Natalia García (PUC, Chile)
- **Título**: Studying algebraic points of bounded degree via algebroid maps.
- **Resumen**: Rational points of symmetric powers of a variety X are closely related to the algebraic points of bounded degree on X. Moreover, proving that the n-th symmetric power of a variety is hyperbolic leads to finiteness of algebraic points of bounded degree by n of the variety, when Lang's conjecture holds.

  In this talk we will study the hyperbolicity level $h(X)$ of a smooth projective variety X, which is the largest n such that the n-th symmetric power of X is hyperbolic, and we will give criteria coming from the (non)existence of non-constant algebroid maps of bounded degree to X. This allows us to give lower bounds for the hyperbolicity level. We will also show that the hyperbolicity level is closely related to well-known arithmetic invariants of curves. This is joint work with Hector Pasten.

(13)
- Tobías Martínez (PUCV-UTFSM-UV, Chile)
- **Título**: The Manin Conjecture for Toric Varieties with few generators over number fields
- **Resumen**: In this talk, we will study Manin's Conjecture on certain projective bundles over projective spaces classified by Kleinschmidt (1988). To do so, we will analyze the analytic behavior of the height zeta function induced by a metrization on the anticanonical bundle, establishing a connection between this height and the norms of hermitian vector bundles over the arithmetic curve induced by the rings of integers of the base number field.

2. Organización sesión

(1) **Martes:**
- Hector del Castillo
- María Chara

- Gustavo Rama

(2) **Miércoles:**

- Ariel Pacceti
- Gonzalo Tornaría
- Natalìa García
- Luis Palacios

(3) **Jueves:**

- David Grimm
- Lucas Villagra
- Tobías Martínez

(4) **Viernes:**

- Harald Helfgott
- Lola Thompson
- Claudio Bravo